Math 243 **Name (Print):** _____
Summer 2018
Practice Exam 2
Doomsday
Time Limit: Probably Not Enough

Problem	Points	Score
1	15	
2	20	
3	20	
4	15	
5	15	
6	20	
7	20	
8	20	
Total:	145	

1. (15 points) Let $f(x) = \frac{1}{16-x^2-y^2}$

   a) Find the domain and range of $f(x)$.

   b) Is the domain open/closed or neither? What is the boundary of the domain? Is the domain bounded or unbounded?

   c) Graph the level curves $f(x) = \frac{1}{\sqrt{15}}$ and $f(x) = 5$. Include the vector $\nabla f|_{(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})}$ on the appropriate level curve.

2. (a) (10 points) Find $\lim_{(x,y)\to(2,2)} \dfrac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$ if it exists.

(b) (10 points) Find $\lim_{(x,y)\to(0,0)} \dfrac{x^4-y^2}{x^4+y^2}$ if it exists.

3. (a) (10 points) Find $\dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial y}\left(\dfrac{y^2+\sin(y)}{y^4}+x\right)\right)$

(b) (10 points) Let $f(x,y,z)=\dfrac{ye^{xyz}}{x}$. Find f_x, f_y and f_z.

4. (15 points) a) Suppose that $r(t) = g(t)\ i + h(t)\ j$ is a vector valued function such that $f(g(t), h(t)) = c$ for some constant c. Show that ∇f and $\frac{dr}{dt}$ are orthogonal along this level curve.

b) Find the the derivative of $f(x, y) = \ln(x^2 + y^2)$ in the direction of $v = i + j$ at the point $(1, 1)$.

5. (15 points) a) Let $z = x^2 - y^2 + 3$. Find the equation of the tangent plane at the point $(1, 1, 3)$.

b) The surface $x^2 + y^2 = 4$ is "sliced" by the plane $x + y + z + 1 = 0$ and forms an ellipse. Find the parametric equations for the tangent line to this ellipse at the point $(2, 2, -5)$.

6. (a) (20 points) Let $f(x, y) = 9x^3 + y^3/3 - 4xy$. Use the second derivative test to find any local min/max or saddle points.

7. (20 points) Find the point on the graph of $z = xy + 1$ that is closest to the origin. (extra credit?)

8. (20 points) Find the cubic approximation for the function $f(x, y) = e^x \ln(1 + y)$ centered at the origin.